[2] $\boxed{4x\frac{dy}{dx} - (1+x\cot x)y = (\sec^2 x)y^5}$ $\boxed{v = y^{1-5} = y^{-4}}$

$$\frac{dv}{dx} = -4y^{-5}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{4}y^5\frac{dv}{dx}$$

$$\boxed{-xy^5\frac{dv}{dx}} - (1+x\cot x)y = (\sec^2 x)y^5$$

$$\frac{dv}{dx} + \left(\frac{1}{x}+\cot x\right)y^{-4} = -\frac{\sec^2 x}{x}$$

$$\boxed{\frac{dv}{dx} + \left(\frac{1}{x}+\cot x\right)v = -\frac{\sec^2 x}{x}}$$

CHECKPOINT: LINEAR

$$\mu = e^{\int(\frac{1}{x}+\cot x)\,dx} = \boxed{e^{\ln|x|+\ln|\sin x|}} = \boxed{x\sin x}$$

$$\boxed{(x\sin x)\frac{dv}{dx} + (\sin x + x\cos x)v = -\sin x\sec^2 x}$$

CHECKPOINT:
$(x\sin x)'$
$= \sin x + x\cos x$ ✓

$$(x\sin x)v = \int -\sec x\tan x\,dx + C$$

$$= -\sec x + C$$

$$\boxed{y^{-4} = v = \frac{C-\sec x}{x\sin x}}$$

$$\boxed{y^4 = \frac{x\sin x}{C-\sec x}}$$

[3] $\boxed{t^2\,dr + (r^2 - rt + 4t^2)\,dt = 0}$

$$\left.\begin{array}{l} (kt)^2 = k^2(t^2) \\ (kr)^2 - (kr)(kt) + 4(kt)^2 = k^2(r^2 - rt + 4t^2) \end{array}\right\} \text{BOTH HOMOGENEOUS ORDER 2}$$

$\boxed{r = vt} \longrightarrow dr = v\,dt + t\,dv$

$\boxed{t^2(v\,dt + t\,dv) + (v^2t^2 - vt^2 + 4t^2)\,dt = 0}$ 1

$v\,dt + t\,dv + (v^2 - v + 4)\,dt = 0$

$\boxed{(v^2+4)\,dt + t\,dv = 0}$ 1

$\boxed{\int \frac{1}{t}\,dt = \int -\frac{1}{v^2+4}\,dv}$ CHECKPOINT: SEPARABLE

1 $\boxed{\ln|t| + C = -\frac{1}{2}\tan^{-1}\frac{v}{2}} = -\frac{1}{2}\tan^{-1}\frac{r}{2t}$

$\boxed{r = 2t\tan(C - 2\ln|t|)}$

ALTERNATELY

GRADE AGAINST ONLY 1 VERSION OF THE SOLUTION — DON'T FORGET THE FIRST 2 ITEMS AT THE TOP OF THE SOLUTION

$\boxed{t = vr} \longrightarrow dt = v\,dr + r\,dv$

$\boxed{v^2r^2\,dr + (r^2 - vr^2 + 4v^2r^2)(v\,dr + r\,dv) = 0}$ 1

$v^2\,dr + (1 - v + 4v^2)(v\,dr + r\,dv) = 0$

$\boxed{(v + 4v^3)\,dr + (1 - v + 4v^2)\,r\,dv = 0}$

$\boxed{\int \frac{1}{r}\,dr = -\int \frac{1 - v + 4v^2}{v(1+4v^2)}\,dv}$ CHECKPOINT: SEPARABLE

$\int \frac{1}{r}\,dr = -\int\left(\frac{1+4v^2}{v(1+4v^2)} - \frac{v}{v(1+4v^2)}\right)dv$

$\int \frac{1}{r}\,dr = \boxed{-\int\left(\frac{1}{v} - \frac{1}{1+4v^2}\right)dv}$

$\boxed{\ln|r| + C = -\ln|v| + \frac{1}{2}\tan^{-1}2v}$

$\ln|r| + C = -\ln\left|\frac{t}{r}\right| + \frac{1}{2}\tan^{-1}\frac{2t}{r}$

$\cancel{\ln|r|} + C = -\ln|t| + \cancel{\ln|r|} + \frac{1}{2}\tan^{-1}\frac{2t}{r}$

$\boxed{C + 2\ln|t| = \tan^{-1}\frac{2t}{r}}$

$\boxed{r = \frac{2t}{\tan(C + 2\ln|t|)} = 2t\cot(C + 2\ln|t|)}$

NOTE: $2t\tan(C-2\ln|t|)$

$= 2t\tan\left(\frac{\pi}{2}-(K+2\ln|t|)\right)$

$= 2t\cot(K+2\ln|t|)$

SO THE 2 ANSWERS ARE EQUIVALENT

$C=\frac{\pi}{2}-K$ OR $K=\frac{\pi}{2}-C$

$\tan\left(\frac{\pi}{2}-x\right)=\cot x$

COFUNCTION IDENTITY FROM TRIG

[4] $\boxed{\underbrace{\left(2x^{n+4}y^{-1}(\ln y)^k - 2x^{n+1}y^{-1}(\ln y)^k\right)}_{P} dy + \underbrace{3x^n(\ln y)^{k+1}}_{Q} dx = 0}$

$$\boxed{\begin{aligned} P_x &= 2(n+4)x^{n+3}y^{-1}(\ln y)^k - 2(n+1)x^n y^{-1}(\ln y)^k \\ Q_y &= 3(k+1)x^n y^{-1}(\ln y)^k \end{aligned}}$$

$$\boxed{2(n+4)=0 \qquad -2(n+1)=3(k+1)}$$

$$n=-4 \qquad 6=3(k+1)$$

$$k=1$$

$$\mu = \boxed{x^{-4}\ln y}$$

$$\boxed{\underbrace{\left(2y^{-1}\ln y - 2x^{-3}y^{-1}\ln y\right)}_{M} dy + \underbrace{3x^{-4}(\ln y)^2}_{N} dx = 0}$$

$$\boxed{\begin{aligned} M_x &= 6x^{-4}y^{-1}\ln y \qquad \text{CHECKPOINT: EXACT} \\ N_y &= 6x^{-4}y^{-1}\ln y \end{aligned}}$$

$$F = \int 3x^{-4}(\ln y)^2\,dx = \boxed{-x^{-3}(\ln y)^2 + C(y)}$$

$$F_y = \boxed{-2x^{-3}y^{-1}\ln y + C'(y) = 2y^{-1}\ln y - 2x^{-3}y^{-1}\ln y}$$

$$\boxed{C'(y) = 2y^{-1}\ln y \qquad \text{CHECKPOINT: ONLY } y}$$

$$C(y) = \int 2y^{-1}\ln y\,dy = \boxed{(\ln y)^2}$$

$$u=\ln y \to \int 2u\,du$$

$$\boxed{-x^{-3}(\ln y)^2 + (\ln y)^2 = C}$$

$$\boxed{(\ln y)^2(x^3-1) = Cx^3}$$